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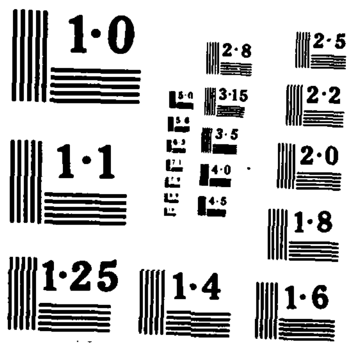
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A NOVEL DIFFERENTIAL GEOMETRIC APPROACH TOWARD ROBUST SIGNAL DETECTION

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ABSTRACT

We present a new approach toward robust signal detection which is based on techniques rooted in differential geometry. These methods, as opposed to the commonly employed classical saddlepoint criteria, readily admit the quantitive measure of the degree of robustness over very general classes of admissable noise distributions. Our approach thus is seen to make possible investigations of the quantitative tradeoff between optimal performance and robustness, and we illustrate the application of this differential geometric approach via various specific examples.

I. Introduction

It is well known that there is increasing interest in the employment of robustness techniques for the discrete time detection of signals in imperfectly known noise. The traditional approach toward addressing questions within this rather broad area of research has been to rely heavily on the classical saddlepoint criterion of Huber (see, for example,[1]). A variety of work appearing in the engineering literature has verified that such an approach can lead to tractable results. However, it may be argued that the degree of robustness obtained owes much to the types of noise models admitted by the method. In reality it may not be easy to verify that the types of models appropriate to the saddlepoint robustness approach sufficiently represent the full extent of variation of the unknown perturbation of a distribution around the nominal. Moreover, although it is possible via the approach of [2]to obtain general representations of the noise model via Choquet capacities [3], it has yet to be seen if such elegant methods are capable of enhancing the denseness of the class of noise models beyond the relatively few standard models (see, for example, [1]). In addition, the saddle-point criterion is inherently a nonquantitative approach toward imparting robustness. We intuitively might suspect that robustness is obtained by a judicious tradeoff with optimal performance, and we thus might desire a way to quantitatively measure the degree of robustness in order that a weighted combination of robustness and performance could be considered subject to some cost criterion. In this paper we present an entirely different approach which views the robustness question not from the saddlepoint perspective but from one which is rooted in differential geometry.

II. Development

Viewing the robustness problem from a slightly different perspective, let $\mathcal{D}_n$ denote the class of n-dimensional distribution functions. From this point of view, the performance of the detector is thus expressed by considering the performance functional $P:\mathcal{D}_n \to R$; we then simply wish to choose the detector so the P is reasonably high and doesn't vary much near the nominal element of $\mathcal{D}_n$. Viewing P as a height function over $\mathcal{D}_n$, we could say that a robust detector would yield a "surface"

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which above the nominal element is both relatively high and not strongly sloped.

Such a perspective thus would indicate that a geometric approach to robust detection might be appropriate. What would be needed would be to provide a differentiable structure to $\mathcal{D}_n$ so that the concept of slope would have the proper meaning. We would then be considering a height function over a differentiable manifold M which would result in a new manifold M_1 for which the Riemannian metric would yield a norm.

In this paper we present some specific applications of the above observations. Noting that a Neyman-Pearson approach involves comparing the sample vector to the appropriate n-dimensional Borel set $\mathcal{B}_n$ in R^n, where n is the number of samples, we then observe that in this robustness application we could in practice regard $\mathcal{B}_n$ as specified via the choice of nominal distribution under H_0; we then would be interested in analyzing the degree of variation in the false alarm probability α and/or detection probability β as the underlying distribution varies about the nominal, thus fixing a choice of height function $h:R^m \to R$ for some natural number m, where $h(\cdot)$ corresponds to the value of α or β for some fixed detector of interest.

Consider first the case where the class of n-dimensional distribution functions is parameterized by m parameters; this class can then be identified with a subset of R^m, and the corresponding manifold M_1 is a surface in R^{m+1}. An appropriate metric tensor $g(\cdot,\cdot)$ is inherited from the standard inner product on R^{m+1} with the obvious choice of coordinate system $\frac{\partial}{\partial y_i} = (\delta_{i1}, \delta_{i2}, \ldots, \delta_{im}, \frac{\partial h}{\partial x_i})$ leading to the components of the metric tensor given by

$$g_{ij} \doteq g\left(\frac{\partial}{\partial y_i}, \frac{\partial}{\partial y_j}\right) = \begin{cases} \frac{\partial h}{\partial x_i} \cdot \frac{\partial h}{\partial x_j} & \text{if } i \neq j \\ 1 + \left(\frac{\partial h}{\partial x_i}\right)^2 & \text{if } i = j \end{cases} .$$

Associating the slope of the unit normal with the cosine of the angle of the unit normal to vertical (with M_1 immersed in R^{m+1}) it is then straightforward, although somewhat lengthy, to show that at the point corresponding to the nominal distribution this cosine is given by

$$\cos \gamma_m = \left(1 + \sum_{i=1}^{m} \left(\frac{\partial h}{\partial x_i}\right)^2\right)^{-1/2} .$$

Note that γ_m provides a measure of local "first order" robustness; smaller values of γ_m suggest less variation in α or β near the nominal distribution.

Consider now the discrete time detection of a constant signal s in additive i.i.d. Gaussian noise with mean μ and variance σ^2. We note that there may be some uncertaintly in all of the values of s, μ and σ^2. Employing first the linear detector, we then choose $h(\cdot)$ to correspond to β and then straightforwardly obtain (for n samples)

$$\frac{d\beta}{ds} = \frac{d\beta}{d\mu} = (n/(2\pi\sigma^2))^{\frac{1}{2}} \exp(-[n(\mu+s)-T]^2/(2n\sigma^2))$$

$$\frac{d\beta}{d\sigma^2} = -(n(\mu+s)-T) \cdot \exp(-[n(\mu+s)-T]^2/(2n\sigma^2)) \ / \ (8\pi n\sigma^6)^{\frac{1}{2}} ,$$

where the threshold T is specified for a given false alarm rate α by evaluating the detector at the nominal values of s, μ, and σ^2. We next employ the robustified version of the linear detector, which replaces the identity function of the linear detector with the nonlinearity $g(\cdot)$ defined by

$$g(x) = \begin{cases} k_2 & \text{if} \quad x > k_2 \\ x & \text{if} \quad k_1 \leq x \leq k_2 \\ k_1 & \text{if} \quad x < k_1 \end{cases} .$$

It is well known that this detector resists the tractable development of closed form expressions for α or β in the Gaussian case. In order to numerically compare the robustness of this "censored" version of the linear detector we therefore employ a large sample Gaussian approximation of the test statistic. The resultant lengthy analysis shows, for example, that with $n=50$, $\alpha=0.05$, $k_1=-0.4$, $k_2=0.6$ and nominally $\mu=0.1$, $s=0.1$, $\sigma^2=1$, we have $\gamma_3=34.3^\circ$, which can be compared to $\gamma_3=68.8^\circ$ for the linear detector.

The robustness of the detector which employs censoring is thus quantitatively demonstrated.

III. The General Case

It would also be important to consider more general classes of distributions; ideally we would wish to place virtually no constraints on the admissable distribution function. Such an approach is actually feasible in the i.i.d. case. Since α and β are expressed via an integral over a Borel set $\mathcal{B}_n$ with respect to the appropriate n-dimensional distribution under H_0 and H_1 respectively, we can without loss of generality note the independence of the observations and investigate perturbations in α and β by limiting consideration to the class of those univariate distributions given by step functions, i.e. those functions of form

$$\tilde{F}(\cdot) = \sum_{i=0}^{m+1} a_i I_{A_i}(\cdot),$$

where the intervals A_i partition R and we take $a_0=0$ and $a_{m+1}=1$. For a fixed finite partition P of R, we note that the corresponding class of step functions can be viewed as parameterized by elements of R^m. Letting $F(\cdot)$ denote the nominal univariate distribution of the observations, we then can employ the aforementioned methods to obtain an expression for $\cos \gamma_m$, where for each partition a Stieltjes approximation to $F(\cdot)$ is chosen and regarded as nominal for the parameterized case. We then define $\gamma = \lim_{|P| \to 0} \gamma_m$, whenever the limit exists. The number γ may be thus be interpreted as the angle of the unit normal to vertical for the general (nonparameterized) distribution case, and as before, we would like this angle to be small for robustness. We also have

$$\cos\gamma = \lim_{|P| \to 0} \left(1 + \sum_{i=1}^{m} \left(\frac{\partial \tilde{h}}{\partial a_i}\right)^2\right)^{-\frac{1}{2}},$$

where, as before, we limit consideration to the situation where the height function $\tilde{h}(\cdot)$ is α or β (with the corresponding univariate distribution of the observations $\tilde{F}(\cdot)$). Now let

$\partial_i^+ \mathcal{B}_n = \{(x_1, x_2, \ldots, x_n)$: there exists $\varepsilon > 0$ such that

$(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n) \in \bar{\mathcal{B}}_n$ for $y \in (x_i - \varepsilon, x_i)$ and

$(x_1, \ldots, x_{i-1}, z, x_{i+1}, \ldots x_n) \in \mathcal{B}_n$ for $z \in (x_i, x_i + \varepsilon)\}$

$\partial_i^- \mathcal{B}_n = \{(x_1, x_2, \ldots, x_n)$: there exists $\varepsilon > 0$ such that

$(x_1, \ldots, x_{i-1}, y, x_{i+1}, \ldots, x_n) \in \mathcal{B}_n$ for $y \in (x_i - \varepsilon, x_i)$

and

$(x_1, \ldots, x_{i-1}, z, x_{i+1}, \ldots, x_n) \in \bar{\mathcal{B}}_n$ for $z \in (x_i, x_i + \varepsilon)\}$

$\partial_{ij}^{++} \mathcal{B}_n = \{(x_1, \ldots, x_{i-1}, x_{i+1}, \ldots, x_n, y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_n)$: there exists w such that $(x_1, \ldots, x_{i-1}, w, x_{i+1}, \ldots, x_n) \in \partial_i^+ \mathcal{B}_n$ and $(y_1, \ldots, y_{j-1}, w, y_{j+1}, \ldots, y_n) \in \partial_j^+ \mathcal{B}_n\}$.

Similarly, we define ∂_{ij}^{+-} by replacing ∂_j^+ in the ∂_{ij}^{++} expression with ∂_j^-. In an analogous manner, we also define ∂_{ij}^{-+} and ∂_{ij}^{--}. We then can establish the following result, which provides a closed form expression for $\cos\gamma$:

__Theorem:__ Suppose that if $x \in R^n \cap \overline{\partial_1^+ \mathcal{B}_n} \cap \overline{\partial_2^+ \mathcal{B}_n} \cap \ldots \cap \overline{\partial_n^+ \mathcal{B}_n} \cap \overline{\partial_1^- \mathcal{B}_n} \cap \overline{\partial_2^- \mathcal{B}_n} \cap \ldots \cap \overline{\partial_n^- \mathcal{B}_n}$ then $x \in \operatorname{int}(\mathcal{B}_n) \cup \operatorname{int}(\bar{\mathcal{B}}_n)$. We then have $\cos\gamma = (1 + \Delta^2 + \Gamma)^{-\frac{1}{2}}$, where

$$\Delta^2 = \sum_{i=1}^{n} \left[\int_{\partial_i^+ \mathcal{B}_n} dF(y_1) \ldots dF(y_{i-1}) dF(y_{i+1}) \ldots dF(y_n) - \int_{\partial_i^- \mathcal{B}_n} dF(y_1) \ldots dF(y_{i-1}) dF(y_{i+1}) \ldots dF(y_n) \right]^2$$

$$\Gamma = \sum_{i \neq j} \left[\int_{\partial_{ij}^{++} \mathcal{B}_n} dF(x_1) \ldots dF(x_{i-1}) dF(x_{i+1}) \ldots dF(x_n) \cdot dF(y_1) \ldots dF(y_{j-1}) dF(y_{j+1}) \ldots dF(y_n) + \right.$$

$$+ \int_{\partial_{ij}^{--} \mathcal{B}_n} dF(x_1) \ldots dF(x_{i-1}) dF(x_{i+1}) \ldots dF(x_n) dF(y_1) \ldots dF(y_{j-1}) dF(y_{j+1}) \ldots dF(y_n)$$

$$-\int_{\partial_{ij}^{-+}\mathcal{B}_n} dF(x_1)\dots dF(x_{i-1})dF(x_{i+1})\dots dF(x_n)dF(y_1)\dots dF(y_{j-1})dF(y_{j+1})\dots dF(y_n) - \int_{\partial_{ij}^{+-}\mathcal{B}_n} dF(x_1)\dots dF(x_{i-1})dF(x_{i+1})\dots dF(x_n)dF(y_1)\dots dF(y_{j-1})dF(y_{j+1})\dots dF(y_n)\Bigg]$$ whenever the integrals exist.

Proof: Recall the step function approximation $\tilde{F}(\cdot)$ to the nominal univariate distribution $F(\cdot)$, $\tilde{F}(\cdot)=\sum_{i=0}^{m+1} a_i I_{A_i}(\cdot)$, where $a_0=0$ and $a_{m+1}=1$. The associated approximate measure of performance is given by $\tilde{h}(\cdot)=\int_{\mathcal{B}_n} d\tilde{F}(y_1)\dots d\tilde{F}(y_n)$, and thus

$$\frac{\partial\tilde{h}}{\partial a_i}=\sum_{i_2=0}^{m}\dots\sum_{i_n=0}^{m} a_1^*(I_{\mathcal{B}_n}(x_{i-1},x_{i_2},\dots x_{i_n})-I_{\mathcal{B}_n}(x_i,x_{i_2},\dots,x_{i_n}))+$$

$$+\sum_{i_1=0}^{m}\sum_{i_3=0}^{m}\dots\sum_{i_n=0}^{m} a_2^*(I_{\mathcal{B}_n}(x_{i_1},x_{i-1},x_{i_3},\dots,x_{i_n})-I_{\mathcal{B}_n}(x_{i_1},x_i,x_{i_3},\dots,x_{i_n}))$$

$$+\dots+\sum_{i_1=0}^{m}\dots\sum_{i_{n-1}=0}^{m} a_n^*(I_{\mathcal{B}_n}(x_{i_1},\dots,x_{i_{n-1}},x_{i-1})-I_{\mathcal{B}_n}(x_{i_1},\dots,x_{i_{n-1}},x_i)),$$

where $a_k^*=(a_{i_1+1}-a_{i_1})\dots(a_{i_{k-1}+1}-a_{i_{k-1}})\cdot(a_{i_{k+1}+1}-a_{i_{k+1}})\dots(a_{i_n+1}-a_{i_n})$ and $x_i=\sup A_i$ for $i=0,1,\dots,m$. Expanding the $\sum_{i=1}^{m}\left(\frac{\partial\tilde{h}}{\partial a_i}\right)^2$ expression, we observe that as the norm of the partition approaches zero, the squared terms collectively reach Δ^2 as a limit, whereas the cross products yield Γ.

QED

We remark that from an intuitive perspective, $\partial_i^+\mathcal{B}_n$ consists of those elements of the boundary of $\mathcal{B}_n$ which are intersected by rays parallel to the i axis and moving in the positive direction from the exterior of $\mathcal{B}_n$ to its interior, whereas $\partial_i^-\mathcal{B}_n$ is formed in an analogous manner with the rays moving in the negative direction from the exterior of $\mathcal{B}_n$ to its interior. In addition, we note that the Theorem's hypothesis is simply a mild condition on the regularity of $\mathcal{B}_n$ which is frequently easy to satisfy in this detection context (wherein $\mathcal{B}_n$ arises by way of a threshold comparator). Moreover, the existence of the integrals in the Theorem is very often easy to verify since the boundary of $\mathcal{B}_n$ is sufficiently well behaved in such cases.

As an example of an application of the Theorem, consider again the linear detector. For n=2 it then follows that (where the detector threshold is T)

$$\partial_1^+\mathcal{B}_2=\partial_2^+\mathcal{B}_2=\{(x,y):y=T-x\},\ \partial_1^-\mathcal{B}_2=\partial_2^-\mathcal{B}_2=\phi$$

$$\partial_{12}^{++}\mathcal{B}_2=\partial_{21}^{++}\mathcal{B}_2=\{(x,y):y=x\}$$

$$\partial_{12}^{+-}\mathcal{B}_2=\partial_{21}^{+-}\mathcal{B}_2=\partial_{12}^{-+}\mathcal{B}_2=\partial_{21}^{-+}\mathcal{B}_2=\partial_{12}^{--}\mathcal{B}_2=\partial_{21}^{--}\mathcal{B}_2=\phi.$$

We therfore have $\cos\gamma=1/(1+1+1+0)^{1/2}=1/3^{1/2}$, i.e. $\gamma=54.7^0$, regardless of the nominal distribution. This may be generalized to show that for n samples $\cos\gamma=(1+n)^{-1/2}$, regardless of the nominal distribution. Note that $\lim_{n\to\infty}\cos\gamma=0$, i.e. the linear detector becomes completed unrobust (as measured by γ) as the number of samples approaches infinity.

On the other hand, consider the classical robustified version of the linear detector, wherein a "censored" detector nonlinearity $g(\cdot)$ of form

$$g(x)=\begin{cases} x & \text{if } |x|\le k \\ k & \text{if } x>k \\ -k & \text{if } x<-k \end{cases}$$ is used.

It then can be shown that

$$\partial_1^+\mathcal{B}_2=\{(x,y):x=T-k,y>k;\ \text{or}\ y=T-x,\ T-k\le x<k\}$$

$$\partial_2^+\mathcal{B}_2=\{(x,y):y=T-k,x>k;\ \text{or}\ y=T-x,\ T-k<x\le k\}$$

$$\partial_1^-\mathcal{B}_2=\partial_2^-\mathcal{B}_2=\phi$$

$$\partial_{12}^{++}\mathcal{B}_2=\partial_{21}^{++}\mathcal{B}_2=\{(x,y):x\ge k \text{ and } y\ge k,\ \text{or}\ y=x \text{ and } T-k<x<k\}$$

$$\partial_{12}^{+-}\mathcal{B}_2=\partial_{21}^{+-}\mathcal{B}_2=\partial_{12}^{-+}\mathcal{B}_2=\partial_{21}^{-+}\mathcal{B}_2=\partial_{12}^{--}\mathcal{B}_2=\partial_{21}^{--}\mathcal{B}_2=\phi.$$

For this case the exact value of $\cos\gamma$

depends on the values of k,T, and the choice of nominal distribution. However, we can make some general conclusions when the amount of censoring approaches maximal ($k \to 0$). In this case we have, when n=2,

$$\lim_{k \to 0} \cos\gamma = (1+2(1-F(0))^2+2(1-F(0))^2)^{-\frac{1}{2}}.$$

For the common case where $F(0) \leq \frac{1}{2}$, we then obtain

$$\lim_{k \to 0} \cos\gamma \leq 1/2^{\frac{1}{2}} \text{ ,i.e. } \lim_{k \to 0} \gamma \geq 45^{\circ},$$

which may be compared to the linear detector's $\gamma=54.7^{\circ}$ for n=2. This may be generalized through a lengthy analysis to conclude that for n samples,

$$\lim_{k \to 0} \cos\gamma = (1+n(1-F(0))^{2(n-1)}+n(n-1)\cdot \cdot(1-F(0))^n)^{-\frac{1}{2}}.$$

Note that for $F(0)>0$ we have $\lim_{n \to \infty}(\lim_{k \to 0} \cos\gamma) = 1$, that is, the detector approaches possessing complete robustness (as measured by γ) as the number of samples tends to infinity. For $F(0) \leq \frac{1}{2}$ we have

$$\lim_{k \to 0} \cos\gamma \leq (1+n\cdot 2^{-2(n-1)}+n(n-1)2^{-n})^{-\frac{1}{2}}.$$

Note that the upper bound can be approached arbitrarily closely for F(0) near ½. For n=3 this upper bound becomes 0.8 (corresponding to $\gamma=36.9^{\circ}$), whereas for n=10 it becomes 0.96 (corresponding to $\gamma=16.3^{\circ}$). This can be compared to the case of the linear detector, where for n=3 we have $\gamma=60^{\circ}$ and for n=10 we have $\gamma=72.5^{\circ}$. For the larger values of n the robustness advantages of the classical robustified linear detector are thus quantitatively demonstrated.

IV. Second Order Robustness

Finally, we note that our geometric approach admits the additional "second order" sensitivity check provided by curvature. There are many different definitions of curvature, however scalar curvature has the advantage of being independent of the local coordinate system employed, thus simplifying its computation. In addition, the choice of scalar curvature is intuitively appealing since it is just a sum of the various sectional curvatures; we are therefore simply accumulating ordinary two dimensional Gaussian curvature it all possible orthogonal directions. For the parameterized distribution case it would then be possible to generate numerical values for the "second order" measure of robustness provided by scalar curvature, which from [4] is given by (where here the Einstein summation convention is used)

$$R_m = g^{ik}\left[\frac{\partial \Gamma^{\ell}_{ik}}{\partial x_{\ell}} - \frac{\partial \Gamma^{\ell}_{i\ell}}{\partial x_k} + \Gamma^{\ell}_{s\ell}\Gamma^{s}_{ik} - \Gamma^{\ell}_{sk}\Gamma^{s}_{i\ell}\right]$$

(m not summed), where the Christoffel symbols Γ^k_{ij} are given by

$$\Gamma^k_{ij} = \frac{1}{2} g^{ku}\left(\frac{\partial g_{ju}}{\partial x_i} + \frac{\partial g_{ui}}{\partial x_j} - \frac{\partial g_{ij}}{\partial x_u}\right) \text{ , in}$$

which $(g^{ij}) = (g_{ij})^{-1}$. Although the above equations regarding scalar curvature appear rather compact, numerical calculations involving them can be quite tedious (but not difficult), especially for large m.

III. Conclusion

We have presented a new approach toward robust signal detection which is based on differential geometric methods as opposed to classical saddlepoint criteria. These techniques are seen to admit a quantitative measure of robustness through the geometric concepts of unit normal slope and scalar curvature, thus allowing the consideration of a weighted combination of performance, first order robustness (via unit normal slope), and second order robustness (via scalar curvature) subject to some cost criterion of interest. Our techniques are additionally illustrated in the paper through various specific examples.

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